**CHAPTER 1**

**EXERCISE 1.1.**  (1) Let $G = (N, A)$ be the graph in which the shortest path problem needs to be solved. Construct a graph $G' = (N, A \cup A_1)$ such that $A_1$ consists of directed arcs from each node in $N - \{s\}$ to the source node $s$ and has both lower and upper bounds equal to unity (see Figure S1.1(a)). We set the cost of every arc in $A$ equal to one and the cost of every arc in $A_1$ to zero. In the optimal minimum cost circulation of $G'$, the arcs in $A$ with nonzero flow will define the shortest paths in $G$.

(2) Let $G = (N_1 \cup N_2, A)$ be the graph in which the assignment problem needs to be solved. Construct the graph $G' = (N_1 \cup N_2 \cup \{p\}, A \cup A_1)$ where $p$ is a dummy node and $A_1$ is defined as $A_1 = \{(i, p) : i \in N_2\} \cup \{(p, j) : j \in N_1\}$. For illustration, see Figure S1.1(b). The lower and upper bounds on the capacity of each arc in $A_1$ is 1. The lower bound on every arc in $A$ is zero and its upper bound is 1. The cost of every arc in $A_1$ is zero.

(3) The construction is exactly similar to that in part (2) except that we set the upper and lower bounds on the capacity of every arc $(p, i)$ and $(i, p)$ in $A_1$ to $|b(i)|$. Further, the capacities and the costs of arcs in the transformed problem are the same as those in the original problem.

**EXERCISE 1.3.** The solution to this exercise is exactly similar to that described in Application 1.2, except that we define the arc cost $c_{ij}$ as:

$$c_{ij} = K_j + c_j \sum_{k=i+1}^{j} \left[ D_k / \left\lfloor \frac{L_i}{L_k} \right\rfloor \right].$$

Note that this solution is based on the assumption that the solution must be *contiguous*, i.e., if length $L_i$ is cut from a beam of length $L_j$ for $i < j$, then lengths $L_{i+1}, L_{i+2}, \ldots, L_{j-1}$ must also be cut from the beam of length $L_j$.

**EXERCISE 1.5.** Construct the bipartite graph $G = (N_M \cup N_W, A)$ in which nodes in $N_M$ correspond to men, nodes in $N_W$ correspond to women, and an arc $(i, j)$ exists in $A$ if and only if the man corresponding to node $i$ is...
compatible with the woman corresponding to node j. Solve a maximum cardinality matching problem in the graph G. The arcs in the matching correspond to the couples in the actual matching.

**EXERCISE 1.7.** Construct a graph having \( n+1 \) nodes, numbered 1 through \((n+1)\). For every pair of nodes \([i, j]\), introduce the arc \((i, j)\) if and only if \(i < j\); let the cost of this arc be \(-c_{ij}\). Find a shortest path from node 1 to \(n+1\). The node numbers lying on the shortest path will denote the word numbers where new lines should begin.

**EXERCISE 1.9.** Construct the graph \( G = (N_S \cup N_P \cup \{p\}, A) \), where every node in \( N_S \) corresponds to a shift, every node in \( N_P \) corresponds to a precinct, \( p \) is dummy node and \( A \) is defined as \( A = N_S \times N_P \cup \{(p, i) : i \in N_S \} \cup \{(j, p) : j \in N_P \} \). Let the lower and upper bounds on the capacities of the arcs in \( N_S \times N_P \) correspond to the minimum and maximum requirement of cars for a particular shift-precinct combination. Further, the lower bound of each arc \((j, p)\) correspond to the minimum number of cars required in the precinct corresponding to node \(j\), and the lower bound of each arc \((p, i)\) corresponds to the minimum number of cars required in the shift corresponding to node \(i\). The cost of every arc emanating from node \(p\) is unity. (Hence the cost of the circulation will be equal to the number of cars committed to the field). All other arc costs are zero. Figure S1.9 illustrates this formulation.

![Figure S1.9](image-url)